# Math III

**The Real Number System**                                                                                    **N-RN**
**Use properties of rational and irrational numbers.**
**N-RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**Quantities**                                                                                                     **N-Q**
**Reason quantitatively and use units to solve problems.**
**N-Q.1**  Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**N-Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**N-Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**The Complex Number System**                                                                                    **N-CN**
**Perform arithmetic operations with complex numbers.**
**N-CN.1** Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.

**N-CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

**Use complex numbers in polynomial identities and equations.**
**N-CN.7** Solve quadratic equations with real coefficients that have complex solutions.

**N-CN.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

**Seeing Structure in Expressions**                                                                               **A-SSE**
**Interpret the structure of expressions.**
**A-SSE.1** Interpret expressions that represent a quantity in terms of its context.
    a.  Interpret parts of an expression, such as terms, factors, and coefficients.
    b.  Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

**A-SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

**Write expressions in equivalent forms to solve problems.**
**A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    b.  Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**A-SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

**Arithmetic with Polynomials and Rational Expressions**                  **A-APR**
**Perform arithmetic operations on polynomials.**
**A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**Understand the relationship between zeros and factors of polynomials.**
**A-APR.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

**A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**Use polynomial identities to solve problems.**
**A-APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

**Rewrite rational expressions.**
**A-APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

**A-APR.7** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

*Note: Limit to rational expressions with constant, linear, and factorable quadratic terms.*

**Creating Equations**                               **A-CED**
**Create equations that describe numbers or relationships.**
**A-CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

**A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

**A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

**Reasoning with Equations & Inequalities** <span style="float:right">**A-REI**</span>

**Understand solving equations as a process of reasoning and explain the reasoning.**
**A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**A-REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

**Solve equations and inequalities in one variable.**
**A-REI.4** Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

**Represent and solve equations and inequalities graphically.**
**A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**A-REI.11** Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**Interpreting Functions** <span style="float:right">**F-IF**</span>

**Understand the concept of a function and use function notation.**
**F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**Interpret functions that arise in applications in terms of the context.**
**F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

**Analyze functions using different representations.**
**F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

**F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

| **Building Functions** | **F-BF** |
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**Build a function that models a relationship between two quantities.**
**F-BF.1** Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

**F-BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

**Build new functions from existing functions.**
**F-BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**F-BF.4** Find inverse functions.
a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 $x^3$ or f(x) = (x+1)/(x–1) for x ≠ 1.*

| **Linear and Exponential Models** | **F-LE** |
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**Construct and compare linear, quadratic, and exponential models and solve problems.**
**F-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

**F-LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

| **Trigonometric Functions** | **F-TF** |
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**Extend the domain of trigonometric functions using the unit circle.**
**F-TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

**F-TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**Model periodic phenomena with trigonometric functions.**
**F-TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

**Prove and apply trigonometric identities.**
**F-TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

| **Congruence** | **G-CO** |
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**Experiment with transformations in the plane**
**G-CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**Prove geometric theorems**
**G-CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

**G-CO.10** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

**G-CO.11** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

**Make geometric constructions**
**G-CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

| **Similarity, Right Triangles, & Trigonometry** | **G-SRT** |
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**Understand similarity in terms of similarity transformations**
**G-SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**G-SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**Prove theorems involving similarity**

**G-SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

**G-SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

| Circles | G-C |
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**Understand and apply theorems about circles**

**G-C.1** Prove that all circles are similar.

**G-C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

**G-C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

**Find arc lengths and areas of sectors of circles**

**G-C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

| Expressing Geometric Properties with Equations | G-GPE |
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**Translate between the geometric description and the equation for a conic section**

**G-GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

**G-GPE.2** Derive the equation of a parabola given a focus and directrix.

| Modeling with Geometry | G-MG |
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**Apply geometric concepts in modeling situations**

**G-MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

| Interpreting Categorical and Quantitative Data | S-ID |
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**Summarize, represent, and interpret data on a single count or measurement variable**

**S-ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

| Making Inferences and Justifying Conclusions | S-IC |
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**Understand and evaluate random processes underlying statistical experiments**

**S-IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

**Make inferences and justify conclusions from sample surveys, experiments, and observational studies**

**S-IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

**S-IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

**S-IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

**S-IC.6** Evaluate reports based on data.

| Using Probability to Make Decisions | S-MD |
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**Use probability to evaluate outcomes of decisions**

**S-MD.6** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

**S-MD.7** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).